Experimental varification of the Empirical law or Wiedemann-Franz law : Kohlrausch method :—

Electrical method is a very simple but accurate method of determining directly the ratio of the thermal conductivity to electrical conductivity. This direct method was given by Kohlrausch and the method was experimentally worked out by Jaeger and Diesselhorst. From the knowledge of K/σ and of the electrical conductivity (σ), 'K' can also be found. The theory of the method may be discussed as below :—

When an electric current passing through a metallic straight bar, it is heated, and according to Joule's law, the amount of heat generated per second is i^2R watts. This heat will flow sideways, as well as longitudinally, but we suppose that loss of heat from the sides is prevented by the guard-ring or some other device and the two ends of the rod are maintained at definite temperature. Then the current passage may be supposed to be parallel to the axis of the rod.

Let us consider an element of the rod of thickness dx at distance x from the left end of the rod where the tempr. in the steady state is T. Then the rate of the heat gain in the element due to heat conduction is

$$AK\frac{d^2T}{dx^2}\,dx$$

$A \rightarrow$ Cross-sectional area of the rod

$K \rightarrow$ Thermal conductivity

The gain in heat due to the electrical energy supplied will be

$$A\sigma\left(\frac{dE}{dx}\right)^2 \cdot dx$$

Heat gained due to the electrical energy supplied.

$$= i^2 R$$

$$R = \rho \frac{dx}{A} = \frac{1}{\sigma} \cdot \frac{dx}{A}, \quad \sigma = \text{specific conductivity}$$

ρ = specific resistance

$$= -\frac{dE}{R}$$ (−ve sign is put because there is fall of Potential along the direction of the current)

$$= \frac{dE}{\frac{1}{\sigma} \cdot \frac{dx}{A}} = -\sigma A \frac{dE}{dx}$$

$$\therefore i^2 R = \left[-\sigma A \left(\frac{dE}{dx}\right)^2 \cdot \frac{1}{\sigma} \cdot \frac{dx}{A}\right] = \sigma^2 A^2 \left(\frac{dE}{dx}\right)^2 \frac{1}{\sigma} \cdot \frac{dx}{A}$$

$$= \sigma A \left[\frac{dE}{dx}\right]^2 \cdot dx$$

In the steady state in case there is no radiation loss from the sides.

$$Ak\frac{d^2T}{dx^2} \cdot dx + A\sigma\left(\frac{dE}{dx}\right)^2 dx = 0$$

$$K \cdot \frac{d^2T}{dx^2} + \sigma\left(\frac{dE}{dx}\right)^2 = 0$$